Western Australian Certificate of Education Examination, 2015

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3C/3D

Section One: Calculator-free

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Student Number: In figures

In words _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33⅓
Section Two: Calculator-assumed	13	13	100	100	66⅔
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2015*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    *   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    *   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free 33⅓% **(50 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 **(6 marks)**

Determine $\dfrac{dy}{dx}$ for each of the following:

(a) $y = \log_5(x^2 + 9)$. (3 marks)

(b) $x = e^{\sin t}$ and $y = e^{\cos t}$ (simplifying in terms of t). (3 marks)

See next page

Question 2   **(5 marks)**

Consider $f(n) = n^2 - 1$ for any positive integer n.

Prove, by the method of exhaustion, that $f(n)$ when divided by four will yield only two of the possible remainders.
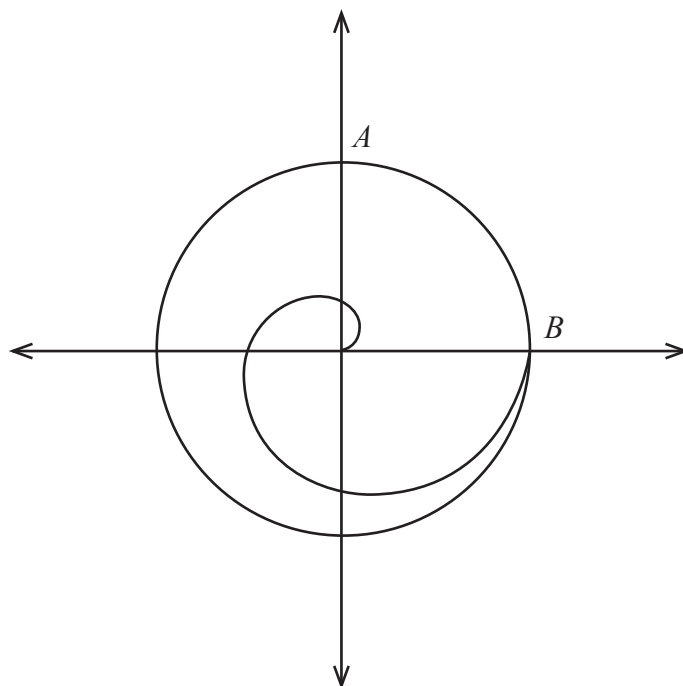
Question 3 **(5 marks)**

The graph of a circle and a spiral is shown in a polar coordinate system.
The circle contains point A with cartesian coordinates $(0, \pi)$.
The spiral intersects the circle at point B, the positive horizontal intercept of the circle.

(a)   Write the coordinates for point A in polar terms $[r, \theta]$.   (1 mark)

(b)   State the polar equation of the circle.   (1 mark)

(c)   Determine the polar equation of the spiral.   (3 marks)

Question 4   **(5 marks)**

It can be shown that $\cos\left(\dfrac{\pi}{8}\right) = \dfrac{\sqrt{2+\sqrt{2}}}{2}$ and $\sin\left(\dfrac{\pi}{8}\right) = \dfrac{\sqrt{2-\sqrt{2}}}{2}$.

(a)   If $\left(\dfrac{\sqrt{2+\sqrt{2}}}{2} + \dfrac{\sqrt{2-\sqrt{2}}}{2}\,i\right)^{7} = a + bi$, determine the values for a and b.   (3 marks)

(b)   If n is a positive integer, for what values of n is $\left(\dfrac{\sqrt{2+\sqrt{2}}}{2} + \dfrac{\sqrt{2-\sqrt{2}}}{2} i \right)^{n}$ a real number?

(2 marks)

See next page

Question 5   **(7 marks)**

Evaluate exactly:

(a) $\int\limits_0^1 \dfrac{1-x}{x+1}\,dx$ (Hint: Let $u = x + 1$)   (4 marks)

(b) $\int\limits_0^{\frac{1}{4}} \cos^2(\pi x)\,dx.$ (3 marks)

See next page

Question 6 **(5 marks)**

(a) Determine $\dfrac{d}{dx}(xe^{2x})$. (2 marks)

(b) Hence evaluate exactly $\displaystyle\int_{0}^{1} 2xe^{2x}\,dx$. (3 marks)

Question 7   **(8 marks)**

The diagram below shows a tangent AB to the circle with centre O and radius r.
Let the size of $\angle BOA$ be θ radians where $0 < \theta < \dfrac{\pi}{2}$.
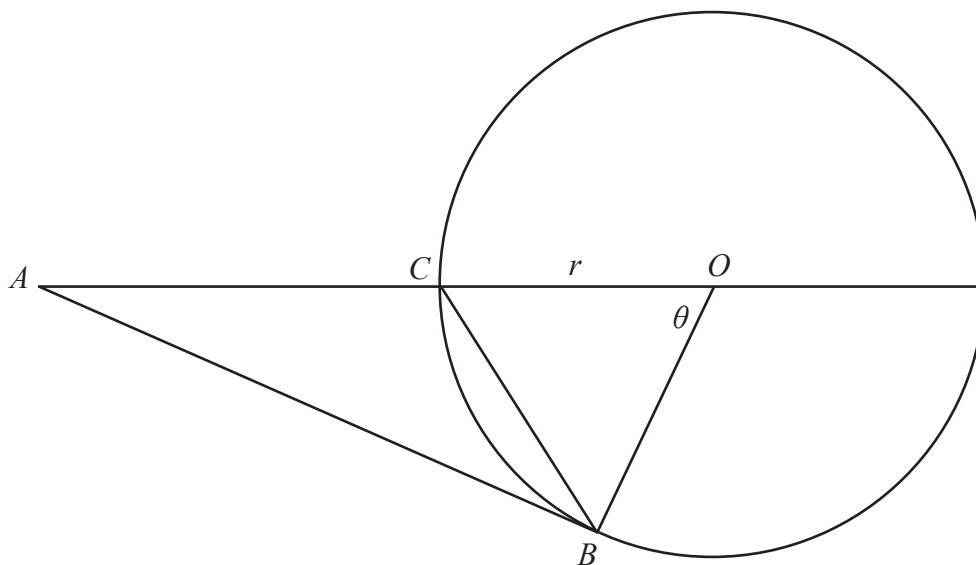
(a)   Determine the area of ΔBOC in terms of r and θ.   (1 mark)

(b)   Determine the area of ΔAOB in terms of r and θ.   (2 marks)

See next page

(c)   Using your results from parts (a) and (b) and the area of sector BOC, show that
$\sin\theta < \theta < \tan\theta$.   (2 marks)

(d)   Using the result from part (c) and by considering limits as $\theta \to 0$ prove that
$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1.$$ (3 marks)

Question 8   **(9 marks)**

The points given by the complex numbers z_1, z_2, z_3, z_4 and z_5 form a regular pentagon in the complex plane such that each complex number is a solution to the equation $z^5 = k$. It is known that $z_5 = 2\,cis\,(-36°)$.
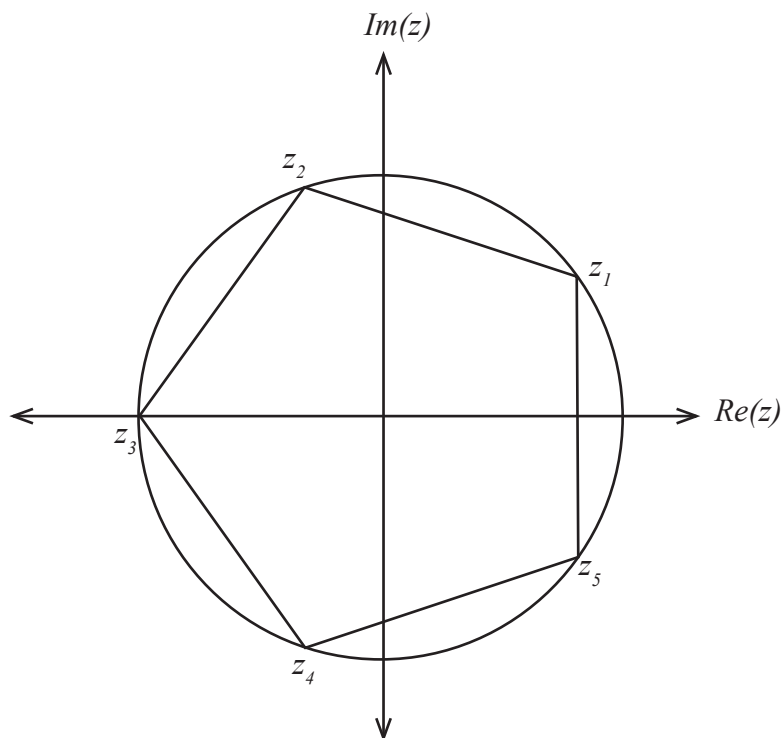
(a)   Determine the value of the constant k.   (2 marks)

(b)   If $z_2 = 2e^{ix}$, determine the exact value for the real number x.   (3 marks)

(c)   Show that $z_4 = \overline{z_2}$.   (1 mark)

It is found that the roots to the equation $z^5 = k$ are such that $z_1 + z_2 + z_3 + z_4 + z_5 = 0$.

Let the complex number $w = e^{i\left(\frac{\pi}{5}\right)}$.

(d)   Prove that $w + w^3 + w^7 + w^9 = 1$.   (3 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____